Study Guide

Inequalities C
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Inequalities - C

An inequality is a number sentence that uses "is greater than," "is greater than or equal to," "is less than," "is less than or equal to," or "is not equal to" symbols. For example, $6n > 4$ is a number sentence with an inequality symbol, and is read "six times n is greater than four." Inequalities can be identified in real world situations by expressions such as "is less than," "is more than," "at least," and "at most." Sentences can be translated to number sentences using the following symbols.

Symbol	Meaning	Associated Word Phrases
$>$	is more than	more than
$<$	is less than	less than
$\geq$	is greater than or equal to	at least
$\leq$	is less than or equal to	at most

An inequality can be solved for a variable (a letter that represents a number) in the same way that an equation is solved. An example of solving an inequality follows.

Example 1: Solve the inequality for x.

$$2x + 7 > 11$$

(1)
$$\begin{array}{r} 2x + 7 > 11 \\ -7 \quad -7 \\ \hline 2x > 4 \end{array}$$

(2)
$$\frac{2x}{2} > \frac{4}{2}$$
$$x > 2$$

Step 1: Subtract 7 from each side of the inequality symbol to isolate the variable term, $2x$, and simplify.
Step 2: Divide both sides of the inequality by 2 to completely isolate x.

Answer: $x > 2$

A compound inequality has more than one condition and can be identified in a number sentence by two inequality symbols, or in a real world application by the words "and" or "or." For example, $3 < x < 9$ is a compound inequality. To read a compound inequality, start in the center and read left: $x > 3$, then go back to the center and read to the right: $x < 9$. The compound inequality is a combination of $x > 3$ AND $x < 9$. Solving a compound inequality is very similar to solving a single inequality.

Example 2: Solve the compound inequality for x.

$$4 < 3x + 1 < 7$$

(1)
$$\begin{array}{r} 4 < 3x + 1 < 7 \\ -1 \quad\quad -1 \quad -1 \\ \hline 3 < 3x < 6 \end{array}$$

(2)
$$\frac{3}{3} < \frac{3x}{3} < \frac{6}{3}$$
$$1 < x < 2$$

Step 1: Subtract 1 from the left, center, and right side of the number sentence to isolate the variable term, $3x$.
Step 2: Divide the left, center, and right side of the number sentence by 3 to isolate x.

Answer: $1 < x < 2$. This answer can be interpreted to mean that x is greater than 1 and x is less than 2.

Although the above examples do not include negative numbers, students must remember to switch the inequality sign when multiplying or dividing by a negative number.
Once the student is familiar with solving compound inequalities, he or she should be ready to solve them in real world situations.

Example 3: The telephone company charges $21.95 per month for basic service plus $0.17 per local call. What is the maximum number of calls that can be made in a month if a family can spend at least $27.50 and at most $32.75 per month for telephone service? Round your answer to the nearest whole number of calls that satisfies the inequality.

(1) $\$27.50 \le \$21.95 + \$0.17c \le \$32.75$

(2)
$$\begin{array}{rrr} \$27.50 \le & \$21.95 + \$0.17c & \le \$32.75 \\ -\ \$21.95 & -\ \$21.95 & -\ \$21.95 \\ \hline \$5.55 \le & \$0.17c & \le \$10.80 \end{array}$$

(3) $\dfrac{\$5.55}{\$0.17} \le \dfrac{\$0.17c}{\$0.17} \le \dfrac{\$10.80}{\$0.17}$

(4) $32.65 \le c \le 63.53$

(5) maximum = 63 calls

Step 1: Translate the information in the problem to a compound inequality. Let *c* represent the number of calls. The family gets charged $21.95 + $0.17*c* each month (the service charge plus local calls). Since the charges are **at least** $27.50 and **at most** $32.75, place the minimum amount of $27.50 to the left of the expression that represents the charges and the maximum amount of $32.75 to the right of the expression. Refer to the chart above to see which inequality symbol to use. Since the question uses the words *at least* and *at most*, the $\le$ needs to be used. Place the $\le$ on either side of the charges expression.
Step 2: Isolate the variable term $0.17*c* by subtracting $21.95 from the left, center, and right of the compound inequality. Simplify.
Step 3: Divide each term of the new compound inequality by $0.17 to isolate *c*.
Step 4: Simplify the inequality.
Step 5: Since the maximum number of calls must be less than or equal to 63.53, the whole number that will satisfy the inequality is 63. Rounding up to 64 would no longer satisfy the inequality because 64 is greater than 63.53.

Answer: 63 calls

Note: If the question had asked what the **minimum** number of calls would be, the answer would be 33, since the minimum number of calls has to be greater than or equal to 32.65.

An activity that can help reinforce the concept of inequalities is to ask the student to make a budget using the amount of money he or she earns each month (approximate or make up an amount if necessary). Then, make up scenarios using the budget. For example:
It costs $120 per month to insure a car, plus $2.05 per gallon of gasoline used. If you have budgeted at least $150.00 and at most $200.00 for car expenses this month, what is the maximum number of gallons of gas you can buy?